$\Lambda = kQ/I$ with I admissible and

write $\mathrm{atom}(I^{\#}) = \{ r_i \mid i = 1, \dots, n \}$

$\rightsquigarrow$ construct a proj. resol of Λ as a right Λ^e-mod.

$$P_2 = \bigoplus_{i=1}^{n} \Lambda e_{s(r_i)} \otimes e_{t(r_i)} \Lambda \xrightarrow{\pi_2} P_1 = \bigoplus_{\alpha \in Q} \Lambda e_{s(\alpha)} \otimes e_{t(\alpha)} \Lambda$$

$$P_1 \xrightarrow{\pi_1} \bigoplus_{i \in Q_0} \Lambda e_i \otimes e_i \Lambda \xrightarrow{m} \Lambda \longrightarrow 0 \qquad (*)$$

we show it's exact.

$m(pe_i \otimes e_i q) = pq$

$\pi_1(e_{s(\alpha)} \otimes e_{t(\alpha)}) = e_{s(\alpha)}\alpha \otimes e_{t(\alpha)} - e_{s(\alpha)} \otimes \alpha e_{t(\alpha)}$

For $r_i \in$ Gröbner basis, write

$$r_i = \sum_{p \in \mathrm{supp}(r_i)} \lambda_p p .$$

$$s(r_i) \xrightarrow{\alpha_{p_1}} \xrightarrow{\alpha_{p_2}} \to \to \to \to \xrightarrow{\alpha_{p_{|p|}}} t(r_i) \quad \text{for } j = 1, \dots, |p|,$$

set $\hat{p}_j = e_{s(r_i)} \alpha_{p_1} \cdots \alpha_{p_{j-1}} \otimes \alpha_{p_{j+1}} \cdots \alpha_{p_{|p|}} e_{t(r_i)}$

$$\pi_2 (e_{s(r_i)} \otimes e_{t(r_i)}) = \sum_{p \in \mathrm{supp}(r_i)} \lambda_p \sum_{j=1}^{|p|} \hat{p}_j$$

we show now exactness of $(*)$.

$$m \circ \pi_1 (e_{s(\alpha)} \otimes e_{t(\alpha)}) = m (e_{s(\alpha)} \alpha \otimes e_{t(\alpha)} - e_{s(\alpha)} \otimes \alpha e_{t(\alpha)})$$
$$= 0$$

so $\mathrm{im}\, \pi_1 \subset \ker m$.

For the converse, Take $X = \sum_{i \in Q_0} \lambda_{p_i q_i} p_i e_i \otimes e_i q_i$

we put an order on

$\bigoplus_{c \in P} \Lambda e_{s(c)} \otimes e_{t(c)}$ where we say

$$p_1 e_{s(c_1)} \otimes e_{t(c_1)} q_1 < p_2 e_{s(c_2)} \otimes e_{t(c_2)} q_2$$

$\iff p_1 c_1 q_1 < p_2 c_2 q_2$ or $p_1 c_1 q_1 = p_2 c_2 q_2$ & $c_1 < c_2$

write $\wedge \operatorname{supp}(x) = p_\wedge \otimes q_\wedge$ where $\deg |p_\wedge q_\wedge| > 0$.

Assume wlog that $p_\wedge = r_\wedge \alpha_\wedge$, $\alpha_\wedge \in Q_1$

$\wedge \operatorname{supp}(X - \pi_1 \lambda_{p_i q_i} r_\wedge \otimes q_\wedge) < \wedge \operatorname{supp}(X)$.

Inductively, $X \in \operatorname{im} \pi_1$.

<u>Now we show exactness $\ker \pi_1 = \operatorname{Im} \pi_2$</u>

$$\pi_1 \left(\sum_{j=1}^{|p|} \hat{p}_j \right) = \sum_{j=1}^{|p|} e_{s(r_i)} \alpha_{p_1} \cdots \alpha_{p_{j-1}} \pi_1 (e_{s(\alpha_{p_j})} \otimes e_{t(\alpha_{p_j})}) \alpha_{p_{j+1}} \cdots \alpha_{p_{|p|}} e_{t(r_i)}$$

$$= \sum_{j=1}^{|p|} e_{s(r_i)} \alpha_{p_1} \cdots \alpha_{p_{j-1}} (\alpha_{p_j} \otimes e_{t(\alpha_{p_j})} - e_{s(\alpha_{p_j})} \otimes \alpha_{p_j}) \alpha_{p_{j+1}} \cdots \alpha_{p_{|p|}} e_{t(r_i)}$$

$$= p \otimes e_{t(r_i)} - e_{s(r_i)} \otimes p.$$

$$\pi_2 \circ \pi_1 (e_{s(r_i)} \otimes e_{t(r_i)}) = \sum_{p \in \operatorname{supp}(r_i)} \lambda_p (p \otimes e_{t(r_i)} - e_{s(r_i)} \otimes p)$$

$$= \left(\sum_{p \in \operatorname{supp}(r_i)} \lambda_p \, p \otimes e_{t(r_i)} \right) - \left(e_{s(r_i)} \otimes \sum_{p \in \operatorname{supp}(r_i)} \lambda_p \, p \right)$$

$$= 0 - 0 = 0$$

we order $\bigoplus_{c \in P} \Lambda e_{s(c)} \otimes e_{t(c)} \Lambda$

$p_1 e_{s(c_1)} \otimes e_{t(c_1)} q_1 < p_2 e_{s(c_2)} \otimes e_{t(c_2)} q_2 \iff$

- $|q_1| > |q_2|$ or
- $|q_1| = |q_2| \wedge |p_1| < |p_2|$ or
- $|q_1| = |q_2| \wedge |p_1| = |p_2| \wedge q_1 > q_2$ or
- $q_1 = q_2 \wedge |p_1| = |p_2| \wedge p_1 < p_2$ or
- $q_1 = q_2 \wedge p_1 = p_2 \wedge c_1 < c_2 \wedge p c_1 q_1 < p_2 c_2 q_2$ or
- $p_1 c_1 q_1 = p_2 c_2 q_2 \wedge c_1 < c_2$.

Jan Marten's order: $i \xrightarrow{\alpha} j \implies i \leq \alpha$

$t(\alpha) < t(\beta)$ or $t(\alpha) = t(\beta) \wedge \left(s(\alpha) < s(\beta) \Rightarrow \alpha < \beta\right)$

Take $x \in \ker \pi_1$ in tensor normal form and

denote $\Lambda \operatorname{supp}(x) = p e_{s(\alpha)} \otimes e_{t(\alpha)} q$, then

$\pi_1(p e_{s(\alpha)} \otimes e_{t(\alpha)} q) = p\alpha \otimes e_{t(\alpha)} q - p e_{s(\alpha)} \otimes \alpha q$ has

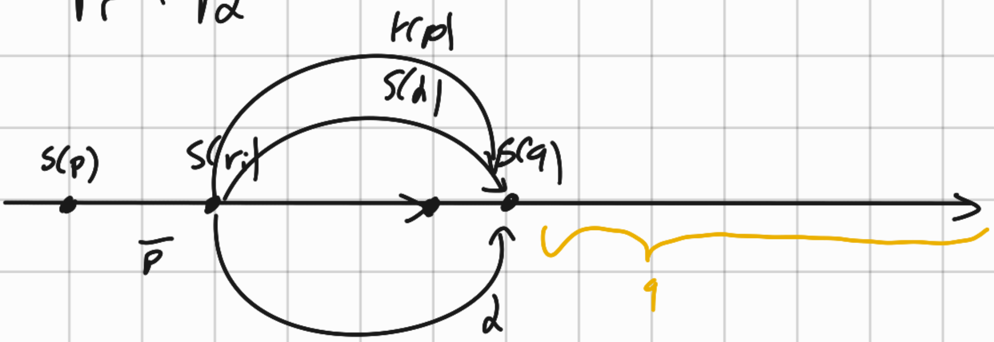
initial monomial $p\alpha \otimes e_{t(\alpha)} q$ and for any other

$p' \otimes q'$ in Supp(X) $\quad p'\alpha' \otimes q' < p\alpha \otimes q$ so

$p\alpha \otimes q$ doesn't reduce over I so there must be

$r_i \in$ atom$(I^{\#})$ s.t. $\exists\, p_r \in$ supp(r_i) with

$p_r \mid p_\alpha$.



$\bar{p}\, p_r = p\alpha$.

Let $p_\wedge = \wedge\,\mathrm{Supp}(r_i)$ then $p_\wedge = p_r$

$$\wedge\,\mathrm{supp}\left(\pi_2\left(e_{s(r_i)} \otimes e_{t(r_i)}\right)\right) = e_{s(r_i)} d_{p_1 1} d_{p_1 2} \cdots d_{p_1 |p_1|-1} \otimes e_{t(r_i)}$$

So $\left(d_{1 p_1} \cdots d_{p_1 |p_1|-1} \otimes e_{t(r_i)}\right)(\bar{p} \otimes q) = p \otimes q$

i.e., $\wedge\,\mathrm{supp}\left(\pi_2\left(e_{s(r_i)} \otimes e_{t(r_i)}\right)\right) \wedge\,\mathrm{supp}\, x = p \otimes q$

$x - \lambda_{pq} d_p^{-1}\left(\pi_2\left(e_{s(r_i)} \otimes e_{t(r_i)}\right)\right)(\bar{p} \otimes q) < X.$